RESEARCH STATEMENT

SCOTT H. MURRAY

1. Overview

My main research interests are **computation in Lie type groups and Kac-Moody groups**, and **algebraic statistics and its applications**. In particular, I work on *representation theory* and *computational group theory*. These are active areas of research, which have great potential for interaction with each other and with other fields.

An algebraic group is a group which is also an algebraic variety. Reductive groups are an important class of algebraic groups: over a finite field they are the finite groups of Lie type and over the real or complex numbers they are Lie groups. The finite groups of Lie type include the bulk of the classes given in the classification of finite simple groups, which in turn are the building blocks of all finite groups. While a great deal is known about how to compute with finite groups and algebraic varieties, relatively little work has been done on computation with algebraic groups and their representations. This has been my focus in the work following my PhD thesis which was on 'Conjugacy classes in maximal parabolic subgroups of the general linear group'.

The methods and techniques I have worked with also extend naturally to infinite dimensional analogues of Lie groups. Kac–Moody groups and algebras are the most natural extensions to infinite dimensions of finite dimensional simple Lie groups and Lie algebras. Affine Kac–Moody groups and algebras have concrete physical realizations and have wide applications in physical theories.

Suitable extensions of the Dynkin diagrams of affine Kac–Moody algebras give rise to hyperbolic and Lorentzian Kac–Moody algebras. The question of associating groups to these algebras is complex, with many of the known constructions not being amenable to computation and applications.

Much of my recent work has involved developing computational methods for Kac–Moody groups. This line of research has been fruitful and has led to some applications in theoretical physics.

AMS subject codes: 20G05, 20–04, 14L17, 20C40, 16G60.

2. Summary of recent work

Constructive homomorphisms for classical groups

Scott Murray and Colva M. Roney-Dougal

In this project, we gave several practical algorithms for computation in simple groups over finite fields and in quasi-simple classical groups that fix a unitary or symplectic form [MR]. Our algorithms were all designed to return objects which are well defined in terms of the input, even if they rely on some randomized steps. We gave algorithms for the following tasks: finding certain canonical non-singular or anisotropic vectors, if the form is nonzero, conjugating the matrix of a form into a canonical (almost) diagonal shape, deciding if two forms are isometric and computing a canonical isometry if one exists, computing the spinor norm of elements in a conformal orthogonal group, constructing certain canonical reflections with respect to a given quadratic form and computing canonical coset representatives.

Prenilpotent pairs in hyperbolic Kac–Moody root systems

Lisa Carbone and Scott Murray

Tits showed how Kac–Moody groups can be presented by generators and relations, generalizing the Steinberg presentation for finite dimensional Lie groups. However the Tits presentation of Kac–Moody groups has infinitely many commutation relations, compared to finitely many in the finite dimensional case, where it is possible to give a commutation relation between every pair of root groups. In the infinite dimensional Kac–Moody case, Tits determined that whenever a pair of real roots is *prenilpotent*, then there is a commutation relation necessary for defining the Kac–Moody group. For rank 2 hyperbolic Kac–Moody algebras, and for the Feingold–Frenkel hyperbolic Kac–Moody algebra AE_3, in [CM], we give a recursive method for determining the prenilpotent pairs of real roots. For AE_3, we determine all real roots that are prenilpotent to a simple root. We give a procedure for determining prenilpotence in any hyperbolic root system whose Weyl group is a free product of finite groups. For E_{10}, recent work of Allcock has shown that the problem of determining all prenilpotent pairs is intractable, but we describe a method to determine if any individual pair of roots is prenilpotent.

Integral group actions on symmetric spaces and discrete duality symmetries of supergravity theories

Lisa Carbone, Scott Murray and Hisham Sati

For $G = G(\mathbb{R})$ a split, simply connected, semisimple Lie group of rank n and K the maximal compact subgroup of G, we gave a method for computing Iwasawa coordinates of G/K using the Chevalley generators and the Steinberg presentation ([CMS]). When G/K is a scalar coset for a supergravity theory in dimensions ≥ 3, we have determined the action of the integral form $G(\mathbb{Z})$ on G/K. As well as allowing us to construct the arithmetic quotient $K\backslash G/G(\mathbb{Z})$, the action of $G(\mathbb{Z})$ on G/K corresponds to 'U–duality' symmetry of the theory. We gave explicit results for the action of the discrete groups $SL_2(\mathbb{Z})$ and $E_7(\mathbb{Z})$ on the scalar cosets $SL_2(\mathbb{R})/SO_2(\mathbb{R})$ and $E_{7(+7)}(\mathbb{R})/[SU(8,\mathbb{R})/\{\pm Id\}]$ for type IIB supergravity in ten dimensions and 11–dimensional supergravity in $D = 4$ dimensions, respectively. This project required new algorithms and new code for infinite root systems. Here we integrated and extended existing Magma code for infinite Coxeter groups.

3. Research description

Chevalley bases for Kac–Moody algebras

Lisa Carbone, Bud Coulson, Shashank Kanade, and Scott Murray

Let $\mathfrak{g} = \mathfrak{g}_{\mathbb{C}}$ be a general Kac–Moody algebra over $\mathbb{C}$. In this project, we define the notion of a *Chevalley basis* for $\mathfrak{g}$. This incorporates the notion of Chevalley basis for affine Kac–Moody algebras that was introduced by Garland and further refined by Mitzman. We define the structure constant $n_{\alpha,\beta}$ for the commutator $[x_\alpha, x_\beta]$ of a pair of root vectors, x_α, x_β whenever $\alpha+\beta$ is a root: real or imaginary. The structure constant $n_{\alpha,\beta}$ is given in terms of the coefficients that occur in the expression of $x_{\alpha+\beta}$ as a linear combination of basis vectors from the $\alpha+\beta$ root space.

We prove that the structure constants with respect to our Chevalley basis are integers and we show that the $\mathbb{Z}$–span of our Chevalley basis is a $\mathbb{Z}$–form of $\mathfrak{g}_{\mathbb{C}}$.

Our methods involve extending a known 'collection algorithm' for writing Lie group commutators for finite dimensional Lie groups in normal form to multibrackets in general Kac–Moody algebras. We describe an implementation of our collection algorithm for Kac–Moody algebras. We give a number of examples, which include finding explicit bases for imaginary root spaces of some affine and hyperbolic Kac-Moody algebras.

The geometry of rank 2 hyperbolic root systems

Lisa Carbone, Scott Murray and Sowmya Srinivasan

Let Δ be a rank 2 hyperbolic root system. Then Δ has generalized Cartan matrix $H(a,b) = \left(\begin{smallmatrix} 2 & -a \\ -b & 2 \end{smallmatrix}\right)$ indexed by $a, b \in \mathbb{Z}$ with $ab \in \mathbb{Z}_{\geq 5}$. If $a \neq b$, then Δ is non-symmetric and is generated by one long and one short simple root, whereas if $a = b$, Δ is symmetric and is generated by two short simple roots. We prove that if a and b are both > 1, then no sum of real roots can be a real root. When a or $b = 1$, we have a strategy to classify all the pairs of real roots whose sum is a real root. We prove that if $a \neq b$, then Δ contains an infinite family of symmetric rank 2 root subsystems $H(k,k)$ for certain $k \geq 3$, generated by either two short or two long simple roots. We also prove that Δ contains non-symmetric root subsystems $H(a',b')$, for certain $a', b' \in \mathbb{Z}$ with $a'b' \in \mathbb{Z}_{\geq 5}$.

Combinatorics of orthogonal arrays with applications to statistics

Scott Murray, Man Van Minh Nguyen and Julio Romero

We have developed an innovative method for finding multi-strength mixed orthogonal arrays via back-track search with symmetry [MN]. These arrays are used as experimental designs in applied statistics. They have useful symmetry and orthogonality properties, and our method is easily adapted to a wide range of specific applications.

The proposed research will apply this to industrial problems. The main idea is to use highly symmetric mixed experimental designs to improve and facilitate decision making mechanisms, as a decision support system, and also as statistical support for Six-Sigma industrial practices. In the mining sector, we expect to improve high-tonnage mobile machine maintenance and troubleshooting, using experimental design to handle reliability, troubleshooting, and maintainability variables. The processes to be enhanced will be taken from Chiles mining industry, due to Julio Romero's extensive experience there.

Input-Output models

Scott Murray and Tai Huu Pham

An Input-Output (IO) Model is a mathematical model for analysing a national or regional economy by estimating flows of intermediate production between sectors. In developed market economies, these models have been superseded by other Computable General Equilibrium Models, but IO Models are still useful for mixed economies because they do not need the assumption that prices are determined by market equilibrium. We are using an IO Model of the Vietnamese economy to estimate the impact of free trade agreements: the ASEAN-Australia-New Zealand Free Trade Agreement and the proposed Trans-Pacific Partnership [PM].

References

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